Unified Engineering Fall 2007
Fluids Problem F15

The F14 class notes summarize the equations and boundary conditions for the potential $\phi(x, y)$ which must hold for an external aerodynamic flow. The corresponding equations and BC's for the streamfunction $\psi(x, y)$ are:

1) at all points in the flowfield: $\nabla^2\psi = 0$ (irrotationality requirement)

2) at all points on the body surface: $\psi = \text{const.}$ (surface flow tangency requirement)

3) at all points "at infinity": $\partial\psi/\partial y = V_\infty$ (consistency with freestream flow)

A horizontal-freestream flow about a circular cylinder of radius R centered on the origin is supposedly given by the following streamfunction:

$$\psi(r, \theta) \;=\; V_\infty \sin\theta \left(r - \frac{R^2}{r} \right) \;+\; 1$$

or equivalently

$$\psi(x, y) \;=\; V_\infty \left(y - \frac{yR^2}{x^2 + y^2} \right) \;+\; 1$$

Does this flow meet the necessary equations and BC's 1,2,3? Verify Yes or No for each.

<u>Useful reference</u>:
Definitions in cartesian and polar coordinates, for any vector field $\vec{F}$ and scalar field f ...

$$\nabla \cdot \vec{F} \;\equiv\; \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \;=\; \frac{1}{r}\frac{\partial (rF_r)}{\partial r} \;+\; \frac{1}{r}\frac{\partial F_\theta}{\partial \theta}$$

$$\nabla \times \vec{F} \;\equiv\; \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \;=\; \frac{1}{r}\frac{\partial (rF_\theta)}{\partial r} \;-\; \frac{1}{r}\frac{\partial F_r}{\partial \theta} \qquad (\hat{k} \text{ component})$$

$$\nabla f \;\equiv\; \frac{\partial f}{\partial x}\,\hat{\imath} + \frac{\partial f}{\partial y}\,\hat{\jmath} \;=\; \frac{\partial f}{\partial r}\,\hat{r} \;+\; \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\theta}$$

$$\nabla^2 f \;\equiv\; \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \;=\; \frac{1}{r}\frac{\partial}{\partial r}\!\left(r\frac{\partial f}{\partial r} \right) \;+\; \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$$